Cooperative Game Theory for Non-linear Pricing of Load-side Distribution Network Support

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1 Introduction

One conventional method for overcoming network line congestion constraint is to employ distributed diesel generators. Although these are costly to operate, they may be less expensive than network upgrades in the short term. An example of a setting where such an option is employed is Bruny Island, Tasmania, Australia. Here, the cost of upgrading the congested line is prohibitive because it is an undersea cable that services a small number of customers. However, the network peak load is exacerbated by the location's popularity as a holiday destination. On the positive side, Bruny Is, like most of Australia, has excellent solar resources. Moreover, in the presence of low feed-in tariffs, combined battery-PV systems installed behind the meter are becoming a viable cost-saving investment for some electricity customers. This represents an opportunity for innovative distribution network service providers (DNSPs) that wish to harness the value of these, and other, distributed energy resources (DER), as a credible alternative option to overcoming a network constraint.[1] Although there are significant engineering challenges to overcome before such a system can be deployed, the question of how to reward DERs participating in such a scheme quickly arises.

Given this context, this paper investigates non-linear pricing for network support provided by behind-the-meter DER such as residential batteries, based on *cooperative game* solution concepts [1–4]. A cooperative game models a problems where a group of players cooperate to earn a joint reward, which has to be apportioned among the players in a fair and stable way. Specifically, these games are used to model strategic situations involving rational, self-interested agents that can form binding contracts with one another to pursue a common action. It is the ability to commit to a course of action that distinguishes cooperative games from the more-widely known *non-cooperative game* formulation.

In particular, we focus on the *Shapley value*, a cooperative value division rule developed in the game theory literature [5–7]. It is implemented as follows: a coalition of agents is paid a fixed sum for satisfactorily completing a task. At the same time, to complete the task, each agent incurs some private cost. Any cooperative solution to the problem allocates payoffs to each agent in such a way that certain desirable criteria are met. For the Shapley value, these criteria are: *efficiency*, the full payment is allocated;

[1] For more detail, see http://brunybatterytrial.org/

symmetry, identical DER are allocated the same amount; *additivity*, if an agent is involved in two separable games at once, its allocation is the sum of its payoffs in the two games separately; and, a *null player* who contributes nothing to the coalition receive zero payoff. In addition, and by design, the Shapley value is unique and always exists, which is uncommon for cooperative solution concepts.

This model and its solution can be straightforwardly mapped to the demand response setting, as follows. A coalition of DERs, probably coordinated by an aggregation service, is paid a fixed sum by a DNSP to provide enough load relief to overcome a predicted thermal constraint excursion. In practise, the battery owners agree to perform some optimal joint network support action using a form of distributed optimisation and control platform, and here we use the approach described in [8].[2] For completing this action, the cooperative system as a whole receives a reward in the form of a payment from the DNSP,[3] which we reasonably expect to be determined by a network monopoly regulating body, such as the Australian Energy Regulator. At the same time, each DER owner incurs some private cost, in the form of energy cost savings foregone, round-trip losses due to charge and discharge inefficiencies, and device degradation. Thus, the players in the cooperative game are DER owners, and the payment to the coalition has to be divided among the DER owners — and for this we investigate the use of the Shapley value.

However, the Shapley value is defined as the average marginal contribution over the set of all possible sub-coalitions of all N participating agents, of which there are 2^N. In general this is an unwhieldy number of computations to make for coalitions of size greater than 25. It is all the more difficult in our distribution network demand response scenario, because each marginal contribution computation involves solving a hard optimisation problem. Accordingly, we turn to approximations of the Shapley value using sampling and search techniques. In particular, attention is directed to the class of *simple games*, which are those with a binary outcome representing the success or failure of a coalition in achieving its task. This binary formulation naturally represents the collective actions of a coalition of battery-owning households that either can or cannot overcome a hard network constraint. By exploiting this structure, we search or sample the graphs of coalition addition operations in an efficient way to quickly produce accurate approximations of the Shapley value.

This paper is structured as follows:

- In the next section, cooperative games are defined, and links to the electricity network support problem domain are made explicit.
- In Section 3, the Shapley value is introduced. We also identify the complications that arise when this solution concept is applied to real-world problems, especially computational issues, and strategies to mitigate them are canvassed.
- Section 4 describes in detail how a cooperative game characteristic function is generated from un underlying optimisation problem. Two network representations are discussed, namely, a copper plate and an single-phase AC network, and the

[2] In the CONSORT project joint actions are computed using network aware coordinate (NAC) algorithms [9], facilitated by Reposit Power's distributed control platform (see: `http://www.repositpower.com/`).

[3] The DNSP in the CONSORT project is TasNetworks.

common features and differences of the characateristic function for these two are identified.
- In Section 5, we present two small demonstrations to illustrate the reward sharing methods that we have developed. This includes a demonstration of a simple sample-based method for approximating the Shapley value. These are small enough to allow explicit demonstration, while allowing us to examine the relationship of the Shapley value to other pricing methods such as locational marginal pricing.
- Section 6 concludes.

2 Cooperative Games

Cooperative games model surplus division problems, in which a group of players that has agreed to cooperate to earn a joint reward also has to decide how it is allocated among them. That is, cooperative game theory is used to analyse which coalitions will form in a given setting, and how a coalition should divide its reward among its members. Thus, the problem one faces in analysing a cooperative game is to find a division of the rewards earned by a coalition that ensures stability of the coalition, or achieves some distributional goals (fairness, proportionality, etc). Here, we deal with an institutional arrangement that proscribes a single coalition forming — this is the coalition contracted to perform the demand response by the DNSP. As such we focus on distributional considerations, and can ignore issues of coalitional stability.

2.1 Preliminaries

We begin with some basic terminology, which can be found in most game theory texts covering cooperative games (e.g. [2]); the foundations of cooperative games are laid out in [1].

If the players in a cooperative game agree to work together, they form a *coalition*. If all N player form a coalition, it is called the *grand coalition*. Each player incurs some private *cost* in completing its component of the joint action, while collectively, the joint action has some *worth* associated with it (i.e. revenue). The difference between the sum of the players costs' and the worth is called the coalition's *surplus*. The task is to divide the surplus in such a way to ensure that the coalition is *stable*. This means that no other smaller coalition could *deviate* to form a new coalition that can also complete the joint action, but divides its surplus in a way that (weakly) improves the allocation to all players in the new coalition. Beyond this broad and loose definition, many refinements to the permissible set of deviations and desirable characteristics of the final surplus division have been proposed, giving rise to a surfeit of *solution concepts*.

Formally, we consider the class of *transferable utility* (TU) games, which are cooperative games that allow payments between players (cf. non-transferable utility games).

Definition 1. *A TU game is given by* $\Gamma = \langle N, w \rangle$ *where:*

- N *is a set of* $|N|$ players*, and*
- $w(S)$ *is a* characteristic function*,* $w : 2^{|N|} \to \mathbb{R}_+$ *with* $w(\emptyset) = 0$*, that maps from each possible coalition* $S \subseteq N$ *to the* worth *of* S*.*

It is important to note that the characteristic function implicitly assumes that the players follow an *optimal* joint action — and in practice computing this optimal action can be difficult.[4]

Also, note that the characteristic function is defined on the powerset of the players, and as such, w can have a very large domain, even for small numbers of players.

2.2 Simple Games

We are interested in a specific class of TU games called *simple games*.

Definition 2. *A* simple *TU game* $\langle N, w \rangle$ *is given by a characteristic function* $w : 2^{|N|} \to \{0,1\}$*, where w is monotonic and* $w(\emptyset) = 0$*. A coalition* $S \subseteq N$ *is* succeeding *if* $w(S) = 1$ *and* failing *if* $w(S) = 0$.

Simple games are typically applied to voting problems, were they are used to measure the relative power of blocks of voters. However, the network support setting we are considering also has a natural simple game structure, where a coalition is succeeding if it is able to overcome the network constraint. Note that this simple structure is dissolved if we move to a probabilistic setting, where there is load uncertainty, and where the coalition may wish to maximise the probability of overcoming the network constraint. On the other hand, this can be turned back into a simple game by assuming a probability threshold needs to be exceeded for the coalition to succeed. Regardless of these details, simple games will form a foundational model for examining Shapley value below.

3 Cooperative Solution Concepts

Solution concepts in cooperative game theory define divisions of the group reward among players, while considering the rewards available to each alternative coalition of players.

3.1 Characteristics of Cooperative Solutions

Before defining the Shapley value, we first formally define some important characteristics of any solution to a TU game.

Definition 3. *Given a* Γ*, a solution concept defines a* payoff *to each player, which is a vector of transfers (payments),* $t = (t_1, \ldots, t_i, \ldots, t_{|N|}) \in \mathbb{R}^I$.

We denote the sum of payoffs as $\sum_{i \in S} t_i = t(S)$. Some desirable properties of solutions concepts include the following; a solution is:

- *Feasible* if $t(S) \leq w(S)$, meaning the total of all the payoffs is less than the coalition's worth,

[4] This is indeed the case in the CONSORT project, where the optimisation problem will ultimately involve solving a three-phase unbalanced multi-period optimal power flow problem. However, this is not the problem tackled in this paper.

- *Efficient* if $t(S) = w(S)$, so that payoff vector exactly divides the coalitions worth,
- *Individually rational* if $t_i \geq w(i)$ for all $i \in N$, meaning the payoff of a player is at least what it can get by acting alone.
- *Symmetric* if $t_i = t_j$ if $w(S \cup \{i\}) = w(S \cup \{j\}),\ \forall\ S \subseteq N \setminus \{i, j\}$. This means that equal payments are made to symmetric players, where symmetry means that we can exchange one player for the other in any coalition that contains only one of the players and not change the coalition's worth. This property is also called *anonymity*, because the players' labels do not affect their payoff.
- *Additive* if for any two additive games the solution can be given by $t_i(v_1 + v_2) = t_i(v_1) + t_i(v_2)$ for all players. That is, an additive solution assigns payoffs to the players in the combined game that are the sum of their payoffs in the two individual games.
- *Zero payoff to a null player* if a player i in w that contributes nothing to any coalition, such that $w(S \cup \{i\}) = w(S)$ for all S, then the player receives a payoff of 0.

We consider the Shapley value solution, although there exist others, such as: the *stable set* [1]; the *core*, with its modern application to cooperative games and present terminology due to [10]; relaxations of the core to the ε-*core* and the *least core* [11]; the *nucleolus* [12, 13]; and *semi-* and *quasi-values*, which relax some technical characteristics of values [14, 15].

Finally, note that we follow Myerson [16, 17] and assume that only one "institutional" coalition may form. Specifically, this is the coalition from which the network purchases the load relief actions, and its players are the only ones that can share the dividend of the cooperative action.

3.2 The Shapley Value

The first and most widely-known value is the Shapley value [5–7].

Definition 4. *The* Shapley value *allocates to player i in a coalitional game $\langle w, N) \rangle$ the payoff:*

$$\phi_i(w) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!\,(n - |S| - 1)!}{n!}\,(w(S \cup \{i\}) - w(S)) \tag{1}$$

Here, the value function ϕ has the following intuitive interpretation: consider a coalition being formed by adding one player at a time. When i joins the coalition S, its *marginal contribution* is given by $w(S \cup \{i\}) - w(S)$. This is the last part of the expression above. Then, for each player, its Shapley value payoff is the average of its marginal contributions over the possible different orders (or permutations) in which the coalition can be formed. In simple games, given one permutation of players, we call the player that makes the coalition successful when it is added the *pivot* of the order.

For example, consider a very simple situation where only three households have batteries installed, but this is enough to overcome the network constraint. This gives rise to eight possible sub-coalitions of battery owners. Some of these sub-coalitions satisfy the constraint, others do not. A fixed operational budget is allocated for paying

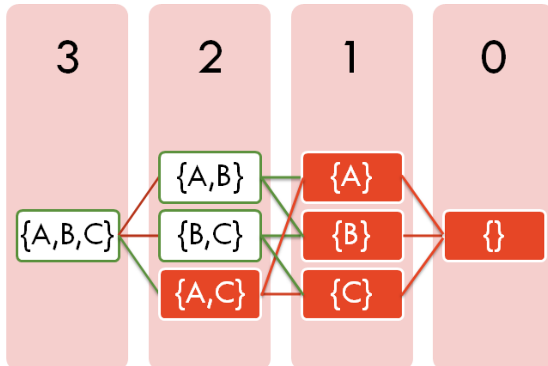

Fig. 1. The coalition graph for $N = \{A,B,C\}$, showing the possible single-agent additions that grow a coalition to form the grand coalition, with the coalition size indicated above.

the battery owners for overcoming the constraint, and the DNSP will allocate the entire budget amount.

This is illustrated in the coalition graph in Fig 1 above, where green boxes show successful coalitions, green lines show marginally successful DER additions. To compute the Shapley value, count the additions that lead to success: $\#A = 1$, $\#B = 4$, $\#C = 1$. These values are normalised according to (1) to get the Shapley value proportions of the DNSP's budget: $\phi = (\frac{1}{6}, \frac{2}{3}, \frac{1}{6})$. So if the DNSP has allocated a budget of $b = \$200$ to overcoming a constraint during a peak event, $t_i = \phi_i * b$, so that A receives \$33, B gets \$134 and C is paid \$33, reflecting the number of coalitions to which their addition makes the cooperative endeavour successful.

The Shapley value is an example of *axiomatic* theory. Specifically, it satisfies the following four properties (defined earlier): (i) efficiency, (ii) symmetry, (iii) additivity, and (iv) zero payoff to a null player. In fact, the Shapley value is the unique map from the set of all games to payoff vectors that satisfies all four properties (i), (ii), (iii), and (iv) above. The combination of properties (ii) and (iii) is often referred to collectively as a *fairness* axiom.[5] When applied to simple games, the Shapley value is often called the *Shapley-Shubik power index* [6], reflecting its use in the analysis of voting power in committees and legislatures.

Some related work is surveyed briefly below. An early application of cooperative game ideas of to the closely related problem of communications network pricing is given in [18]. A more recent effort links cost to an upper percentile contribution to communications network load [19]. Recent applications of cooperative games to problems in power systems can be found in [3, 4]. More generally, Moulin has a series of papers on the topic of fair cost and surplus allocations, especially with respect to capacity network problems and congestion, including [20, 21]. Of particular relevance, [22] apply the Shapley value to computing demand response payments, but their model

[5] Also, in games where the core is not empty, the Shapley value is in the core, and can be considered the centre of gravity of the core.

is generic and not applied to specific power system problem. In contrast we build our cooperative model up from an explicit network optimization problem.

3.3 Computation and Approximations

As noted above, approximations are required for computing the Shapley value, because the number of sub-coalitions to evaluate increases exponentially with N; that is, as the number of combinations of players in the coalition graph grows rapidly. Moreover, exact computation requires enumerating all sub-coalitions, but this is prohibitively expensive for even moderate-sized N, and more so when each sub-coalition worth is itself expensive to compute (as is the case for the power flow-based evaluations underlying the NAC algorithms undertaken in the CONSORT project, which is the application that motivates this work). In order to overcome this challenge, we have developed strategies for reducing the computation burden of computing payoffs to participating DERs. The motivating works for these approaches are stated briefly below, with a more complete treatment left for the full length paper.

To begin, a recent overview of standard approaches to computing solutions to cooperative games is given in [23]. In particular, reinforcement learning and sample-based approaches to estimating the payoffs are being employed in our work. A early Monte Carlo approach to computing the Shapley value in simple games is the Mann-Shapley algorithm [24]. Randomised approaches to computing the Shapley and other power indices in simple games were analysed by [25], and proved that randomly sampling permutations and averaging the marginal contributions constitutes an unbiased estimator of the agents' true Shapley values. A more recent example of this approach in the context of general monotonic cooperative games can be found in [22]. For monotonic game cases, we are developing more sophisticated search and exploration methods, including methods from multi-armed bandit problems [26] and search methods that exploit the monotonicity of the network support game's characteristic function, following [27, 28].

We now show how to derive the characteristic function for the cooperative problem of DERs that are used for network load relief.

4 Optimization-based Cooperative Game

The problem we tackle is to use residential batteries to overcome a network thermal constraint, and to pay them according to their value to the DNSP. In this section, we first define the specific underlying optimisation problem, and describe the way counterfactual cases are computed, thereby defining the characteristic function of the cooperative game model. Ultimately, the characteristic function is used to calculate the *marginal contributions*, $w(S \cup \{i\}) - w(S)$, in (1).

4.1 Constrained Network Optimisation Model

We model the collective problem as using N batteries to solve a *quadratic program* (QP) that minimises the sum of squared power flows through the constrained network element around the time of the expected peak. In practise, this smooths the load during the control period, with the quadratic penalty placing greater emphasis on trimming peaks than lowering the average load. This typically results in a load with a buffer is between the predicted peak flow and the constraint limit, and also allows us to use the computational power of commercial mathematical programming solvers. An alternative approach would be to minimise the maximum power flow through the constrained line directly. However this approach requires the use of variational inequalities or other, more complicated, problem formulations that are not as efficiently solved as one with a quadratic objective, or to rely on hueristics. Given that our ultimate goal is computation of the Shapley value, which requires a very large number of these problems to be solved, we adopt the quadratic programming approach detailed below.

Specifically, at the time of the peak, we expect $S = N$ so that all available batteries participate in overcoming the constraint. In more detail, a grand coalition N collectively minimises the following objective:

$$\min_{\Omega} \sum_{k \in K^*} x_k^2 \tag{2}$$

where x_k is the power flowing through the constrained network line during time slot k, and Ω is the set of control variables. In this paper, the (not yet explicitly stated) control variables are the battery change/discharge decisions, but in general, this set may also include other automated or human-controlled devices, such as hot water cylinders, clothes dryers and air conditioners. The time slots $k \in K^*$ indicate times of the expected peak, possibly non-contiguous, and perhaps with some buffers to prevent a "rebound" peak also overwhelming the constraint.

Minimisation of this objective is subject the operating and energy balance constraints of the flexible load and storage devices in question, and also other pertinent physical network contraints. In more detail, the coupling of all loads and controllable devices at time slot k in order to compute the value of x_k is formulated differently for different models of the network. Here we describe both copper-plate and a single-phase AC network representations, but our results are only for the copper-plate network.

In the simpler case, we assume a copper-plate model of the network below the congested line. Given this, the coupling constraint linking all loads may be given by the linear constraints, for each $k = 1, \ldots, 24$:

$$x_k = x_k^{\mathrm{uc}} + \sum_{i \in N} x_{i,k} \tag{3}$$

where x_k^{uc} is the background uncontrolled load of all customers not participating in the DR scheme, while for each $i \in N$,

$$x_{i,k} = x_{i,k}^{\mathrm{uc}} - x_{i,k}^{\mathrm{pv}} + x_{i,k}^{\mathrm{b}} \tag{4}$$

is the net load of household i, that is, uncontrolled load, $x_{i,k}^{\mathrm{uc}}$, less local PV generation, $x_{i,k}^{\mathrm{pv}}$ plus (minus) battery charge (discharge), $x_{i,k}^{\mathrm{b}}$. A complete description of this formulation of the problem is omitted for brevity, but can be found in [8], where the constraints used to model the batteries include binary variables are used to capture differences in charge and discharge efficiencies, which are themselves treated as constants. However, because these are linear constraints, as are all other constraints in the problem formulation, the optimisation problem is a *mixed-integer quadratic program* (MIQP), which may be solved efficiently using many commercial and open-source mathematical programming solvers. [6]

In the AC power flow case, the customers are connnected at a range of load buses, and likewise, the background load is apportioned across them. Then load at the constrained line is coupled to the net customer and background uncontrolled loads through a set of non-linear constraints capturing the effects of losses and voltage and power angle differences across the lines, etc, of the network. Accordingly, this problem is a *non-linear mixed integer program* NLMIP. The presence of additional network constraints may affect the feasible set of DER control options, for example, by limited a battery's power output when a bus voltage is at its upper limit. Again, for sake of brevity and to maintain our focus on the cooperative game we a deriving, we refer the interested reader to [29] and the references therein for ways to efficiently formulate the AC network representations and solve instances of this class of OPF problem. Nontwithstanding that, as is well understood, these network details considerably complicate computation. Nonetheless, we are able to solve these problems using commercial and open-source non-linear program solvers. Note that the customer net-load copuling constraint (4) remains the same in both formulations.

In the remainder, we refer to the two network model cases above as the *copper-plate* and *AC power flow* models, and their corresponding formulations as MIQP and NLMIP, respectively.

4.2 Computing Counterfactual Load Profiles

When computing the counterfactual effects of fewer batteries contributing load relief to the network, which are needed for calculating the Shapley value, we have $S \subset N$ strictly. In these fictitious cases, we recompute problem (2) for S, subject now to an appropriately altered set of coupling constraints. In particular, this does not mean that the uncooperative batteries can be ignored. Instead, we must also model their self-interested behaviour around the time of the peak load. Specifically, we do this by treating them as individuals, following Myerson, that each separately maximise the value of operating their batteries by load-shifting.

For this, we use a *mixed integer linear program* (MILP), to simulate the individual batteries contribution to the peak, each with an objective given by:

$$\min \sum_{k=1}^{24} p_{i,k}\, x_{i,k}^{+} \tag{5}$$

where the operating and energy balance constraints of each of the storage devices are the same as in the MIQP above. Here, $x_{i,k}^{+}$ are the positive values of (4) across the entire day, multiplied by a time-of-use charge at rate $p_{i,k}$. This represents the battery owner using its device to minimise its costs by maximising self-consumption of locally generated

[6] Note in [8], the MIQP captures the typically increasing marginal costs of running a conventional generator or diesel genset, while here we use the quadratic objective purely as a penalty function that drives load below the thermal power limit across the time interval of interest.

PV and/or by pre-charging the battery from the grid to power it during peak pricing periods. This objective is used to approximate the effects of very low feed-in-tariffs in Australia (5-8c/kWh vs 20-60c/kWh retail) by assuming they are not worth anything at all.

Thus, to generate a characteristic function for the cooperative game model (in order to calculate the Shapley value), we must compute the counter-factual power flows, $x_k(S)$, that would have occurred if some agents $j \in N \setminus S$ were excluded from cooperating in the demand response scheme. To do this, the problem in (5) is solved for each non-participating individual and the power they draw from the network, $x^{\mathrm{LP}}_{j,k}$. In the copper-plate case, the non-participants' loads are added into the coupling constraint in (3) in the same way as background load; that is, for each $k = 1, \ldots, 24$:

$$x_k(S) = \sum_{i \in S} \left(x^{\mathrm{uc}}_{i,k} - x^{\mathrm{pv}}_{i,k} + x^{\mathrm{b}}_{i,k} \right) + x^{\mathrm{uc}}_k + \sum_{j \in N \setminus S} x^{\mathrm{LP}}_{j,k} \tag{6}$$

Then the MIQP is solved for the coalition S using the device contraints for $i \in S$ and the coupling constraints above.

In the AC power flow case, the load profile $x^{\mathrm{LP}}_{j,k}$ computed in the same way, but it is added to the load bus corresponding to the customer's location. The new NLMIP is then solved for the coalition S using the device contraints for $i \in S$ and the existing network power flow constraints.

4.3 The Cooperative Characteristic Function

Finally, given a thermal constraint encoded as a 30 minute-hour energy limit, $E^{\max}$, the worth of a coalition, S, is given by the threshold function:

$$w(S) = \begin{cases} 1 & \text{if } x_k(S) < E^{\max} \text{for all } k = 1, \ldots, 24 \\ 0 & \text{if } x_k(S) \geq E^{\max} \text{for any } k = 1, \ldots, 24 \end{cases} \tag{7}$$

Taken as a whole over all $S \subseteq N$, the threshold function (7) and the underlying optimisation problems define the simple characteristic function for the cooperative game model, and are used to compute the Shapley value.

4.4 Sample-based Approximation of the Shapley Value

For large numbers of agents, exact computation of the Shapley value becaomes intractable. Even for the simplest of simple characteristic functions, such as arise in weighted voting games, exactly computing the Shapley value for more than 25 agents is close to intractable in any reasonalble amount of time. However, for these settings, sample-based approximations of the Shapley value have considerable appeal. While early demonstrations of this approach demonstrated its usefulness [24], here, we rely on a simple sampling scheme, which was recently shown to provide an unbiased estimator of the Shapley value in all simple games [25].

In more detail, the procedure begins by initialising the (vector) estimate of the Shapley value to zero for each agent. Each sample then involves randomly sampling a permutation of agents, and finding the pivotal agent for that permutation. Note that this step can require solving up several of the underlying schedule optimisation problems. Once the pivotal agent it found, it has a value of 1 added to its entry in the value estimate. Finally, after a sufficiently large number of samples, the vector is normalised to sum to one, with the result providing an unbiased estimate of the Shapley value.

In the demonstration section below, we use this simple procedure to estimate the Shapley value for the cooperative load relief problem with 12 agents.

5 Demonstration

In this section, the Shapley value is used to provide value-reflective non-linear pricing to battery owners participating in a load relief demand response scheme. Specifically, we present the results of two small, preliminary tests of the pricing method, and discuss their significance. We also p[ovide some prelimary evidence supporting the use of sample-based approximation methods. The results here only regard the coppoer-plate model; while a key element of our future work will be to explore efficient ways to compute the Shapley value in the AC power flow case, where the underlying optimisation problem is significantly more difficult to solve.

5.1 Scenario Description

In both scenarios, DER agents control batteries with two different capacities and powers, 5kWh/2.5kW or 10.5kWh/5kW. Each household also has a unique uncontrolled load profile and residential rooftop PV generation profile, using data collected by Ausgrid, a distribution network company in Sydney, Australia, and surrounds.[7] These values are incorporated into the coupling constraints (3). The background load on the network is given by historical NSW system load for Friday 10 February, 2017, which was a system peak requiring significant load curtailment to resolve a supply shortfall.[8] In all cases, the constrained line limit is 135kW, which is exceeded by 4.6kW at uncontrolled the peak.

5.2 Demonstration Results and Discussion

The Shapley value is computed for three problems. Problem 1 contains 3 agents, Problem 2 has 12, and Problem 3 has 20.

Problem 1: Three agents In this scenario, two customers, agents 1 and 3, have the smaller 5kWh/2.5kW systems, agent 2 has a 10.5kwh/5kW system. Due to the small size of this problem, exact computation of the Shapley value is possible, so approximation is not required.

[7] Available from: http://www.ausgrid.com.au/Common/About-us/Corporate-information/Data-to-share/Solar-home-electricity-data.aspx

[8] This data is available from the Australian Energy Market Operator: https://www.aemo.com.au/Electricity/National-Electricity-Market-NEM/Data-dashboard#aggregated-data

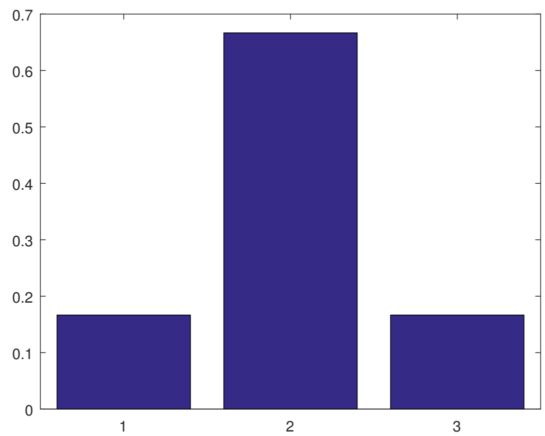

Fig. 2. Shapley value for three agents in Problem 1.

The values to each agent participating in the DR scheme primarily reflects their battery capacity and power. This is seen in Fig. 2, and can be interpreted as meaning that adding Agent 2 is pivotal to four out of six permutations (orderings) of the three players, while Agents 1 and 3 are pivotal in only one of six orders each. Also note that although there is some variability in baseline energy use between the agents, this does not affect their reward according to the Shapley values.

The network profiles for different sub-coalitions around the time of the congestion are plotted in Fig. 3. These show that three sub-coalitions are successful, which corresponds to the computed ϕ.

Problem 2: Twelve agents In problem 2, three agents, 1, 3 and 5, have the smaller 5kWh/2.5kW battery systems, while the remainder have the larger 10.5kwh/5kW systems. Although exact computation of the Shapley value is possible for this setting, it is very time consuming. Accordingly, we use it as a good case to examine and compare exact and approximate methods.

Results for both approaches are shown in Fig. 4, where the bars indicate the exact values, and the points are estimtes with error bars indicating two standard deviations around the mean.

We first consider the exact values. Again, the values to each DER agent reflect their battery capacity and power, with the three lower valued agents having the smaller systems, as illustrated in Fig. 4. However, note that although the agents have some variability in their baseline energy use, this does not appear to affect the exact values. Also, compared to Problem 1, we observe that the values of the agents controlling high- and low-power batteries have less variation between them when there are more agents involved. This reflects the fact that as the number of agents becomes very large, the Shapley value tends to competitive market price uniformly for all agents. Nonetheless,

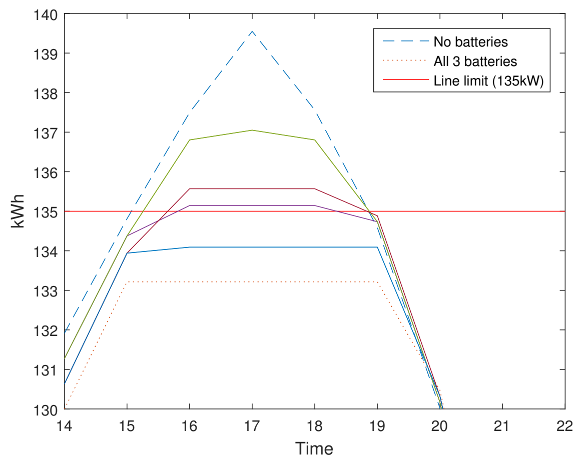

Fig. 3. Load profiles for optimal load relief by all six sub-coalitions in the copper-plate model, with the line limit indicated.

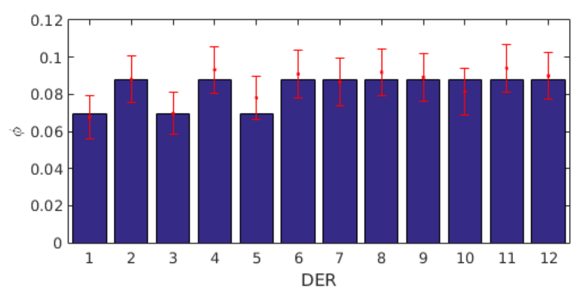

Fig. 4. Shapley value for twelve agents in Problem 2. Bars are the reults from exact computation of the value; while points with error bars indicate the values approximated from 2000 permutation samples.

in the future, when we move to explicitly consider AC network losses and other line constraints, we expect to act as new sources of variation between the agents" Shapley values. This is akin to the way location marginal prices are generated from Lagrange multipliers of constraint equations in optimal power flow problems.

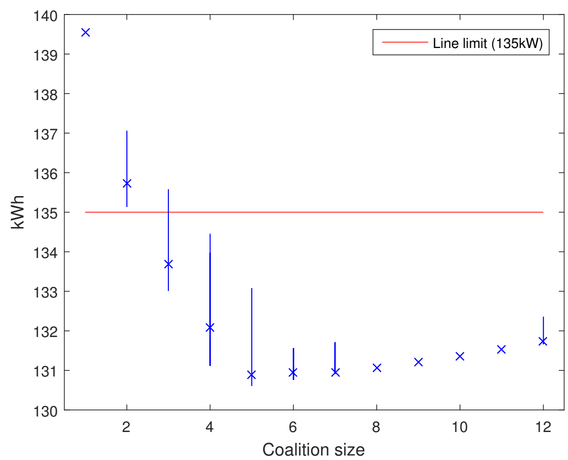

Fig. 5. Plot of the maximum hourly average power vs coalition size for Problem 2. The lines indicates the range of values, and crosses mark the mean for each coalition size.

Fig. 5 shows how the coalition size affects the performance of the load relief system (based on the results for exact computation). From this, see that all coalitions of size 4 and greater are successful. Note that the slow trend up for larger coalition sizes appears to be due energy consumed by the inefficiencies of battery storage.

We now consider the approximate values, generated from 2000 samples of permutations of agents, and indicated by the red points and error bars on Fig. 4. Most importantly, the point estimates and errors are largely consistent with the values computed by the exact computation. This is evidence that the variability around the exact values is due to sampling errors rather than any systematic problem with the approximation procedure. However, is should be clear that 2000 samples is insufficient to statistically discrimiate between the values of DER agents with higher- and lower-powered batteries. As such, one aspecct of our future work is to investigate the required sample size to provide the requisite degree of accuracy in the estimates to make such a discrimiation possible.

Comparison to Lagrange multipliers The results above illustrate why simple linear pricing based on incremental contributions, as given by Lagrange multipliers, to overcoming the line constraint are not appropriate when more than enough battery support is contracted or delivered. Specifically, in Problem 2, the Lagrange multipliers of the line constraints in the MIQP with twelve agents is zero. Thus, by this pricing the constraint using Lagrange multipliers, each would be paid nothing. This is clearly inappropriate way to reward battery owners for delivering load relief.

6 Summary

We investigated non-linear pricing for network support provided by behind-the-meter DER, such as residential batteries, based on the Shapley value solution. In doing so we demonstrated how an electricity network support problem is used to generate a cooperative game characteristic function from the bottom up. We identified the complications that arise when the Shapley value is applied to real-world problems, especially computational issues, and strategies to mitigate them are canvassed. Finally, we illustrated how a characteristic function is computed from a concrete problem of using batteries for load relief in the presence of line congestion, in order to calculate the Shapley value. We then presented results for two small demonstrations, to illustrate the usefulness of the Shapley value in these domains. These problems are small enough to allow explicit demonstration, while allowing us to examine the relationship of the Shapley value to other pricing methods. In particular, we illustrated why incremental cost-based methods, such as locational marginal pricing, are not appropriate for computing payments to battery owners when excess capacity is purchased or supplied. In contrast the Shapley value is an ideal method for computing payments in these settings.

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